Cosmology based on $f(R)$ Gravity admits 1 eV Sterile Neutrinos

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It is shown that the tension between recent neutrino oscillation experiments, favoring sterile neutrinos with mass of order of 1 eV, and cosmological data which impose stringent constraints on neutrino masses from the free streaming suppression of density fluctuations, can be resolved in models of the present accelerated expansion of the Universe based on $f(R)$ gravity.

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The possibility of the existence of light sterile neutrinos has been suggested by neutrino oscillation experiments such as the Liquid Scintillator Neutrino Detector (LSDN) [1] and the Booster Neutrino Experiment (MiniBooNE) [2]. Recent nuclear reactor experiments also favor the additional neutrinos with mass of this range [3]. Further analysis shows increasing experimental evidence that there may exist one or two species of sterile neutrinos with mass of order of 1 eV [4–6].

From the cosmological point of view, recent reanalysis of the primordial helium abundance produced at the big bang nucleosynthesis (BBN) also favors the existence of extra components of radiation [7]. In terms of the effective number of neutrinos, which is defined by the total energy density of the radiation as

$$\rho_r = \rho_\gamma \left[1 + N_{\rm eff} \frac{7}{8} \left(\frac{4}{11} \right)^{4/3} \right] \tag{1}$$

with ρ_γ being the energy density of photons, they find $N_{\rm eff} = 3.68^{+0.80}_{-0.70}$ (2σ) or $N_{\rm eff} = 3.80^{+0.80}_{-0.70}$ (2σ) for the neutron lifetime $\tau_n = 885.4 \pm 0.9$s or 878.5 ± 0.8s, respectively. Note that the standard three flavor neutrino species gives $N_{\rm eff} = 3.046$ [8]. Furthermore, the cosmic microwave background (CMB) anisotropy observations at the small angular scales yield similar value, $N_{\rm eff} = 4 - 5$ [9–12]. Thus both BBN and CMB suggest there exists extra relativistic species, which may be sterile neutrinos that are expected to be thermalized in the early Universe due to mixing [13].

If we further incorporate the large-scale-structure (LSS) data in the cosmological analysis, however, it turns out that in the standard flat Λ cold-dark-matter (ΛCDM) model the sterile neutrino mass is constrained to be appreciably smaller than 1 eV [14–16] to avoid suppression of small-scale fluctuations due to free streaming.

Hence apparently there exists some tension between experimental data of neutrino oscillations and cosmological data. But of course they should not be treated on equal footing because the latter requires a number of assumptions on the cosmic evolution. Indeed there have been some attempts to make cosmology compatible with these experimental data [17, 18] by adopting a wCDM model to treat the equation-of-state parameter of the dark energy, w, as an additional fitting parameter, or by introducing extra radiation components besides the sterile neutrinos. However, the former results in $w < -1$ and larger CDM abundance with the cosmic age significantly smaller than the standard value, while the latter solution may be in conflict with the aforementioned constraint from BBN. So neither of them is an attractive solution.

In this *Letter*, we show that extra growth of small-scale fluctuations at recent redshifts which occurs in viable cosmological models of the present accelerated expansion of the Universe (in other terms, in models of present dark energy) based on $f(R)$ gravity [19–21] can make eV-mass sterile neutrinos compatible with cosmological observations under the proper choice of the function f. $f(R)$ gravity is a simple generalization of General Relativity (GR) obtained by introducing a phenomenological function of the Ricci curvature R, see *e.g.*, the recent review [22]. It represents a special case of more general scalar-tensor gravity with the Brans-Dicke parameter $\omega_{BD} = 0$, and it has an extra scalar degree of freedom (or, scalar particle dubbed scalaron). However, in contrast to Brans-Dicke gravity, scalaron is massive and its rest mass M_s depends on R , *i.e.*, on background matter density in the regime of small deviations from GR. Such models explain the present cosmic acceleration without introducing of a cosmological constant, mathematically this means that $f(0) = 0$. For the $f(R)$ models of present dark energy constructed in [19–21] which satisfy all existing observational data, the deviation of the background evolution from the standard ΛCDM model is small, less than few percent (see *e.g.*, [23]), though not exactly zero. Their most dramatic difference from the standard model appears in the enhancement of the evolution of matter density perturbations on scales smaller than the Compton wavelength of the scalaron field that occurs at redshifts of the order of a few depending of the scale. This extra growth can compensate the suppression

due to the free streaming of massive neutrinos and thus the upper bound for neutrino mass is relaxed in $f(R)$ gravity [24]. We show that the same mechanism works in the case of sterile neutrinos, too, to make cosmology with them compatible with neutrino experiments.

$f(R)$ gravity is defined by the action,

$$S = \frac{1}{16\pi G} \int d^4x \sqrt{-g} f(R) + S_m, \tag{2}$$

where S_m is the action of the matter content and it is assumed to be minimally coupled to gravity (we put $\hbar = c = 1$). If we set $f(R) = R - 2\Lambda$, it reproduces GR with a cosmological constant. Instead, for definiteness we use the following form [21]

$$f(R) = R + \lambda R_s \left[\left(1 + \frac{R^2}{R_s^2} \right)^{-n} - 1 \right], \tag{3}$$

where n, λ, and R_s are model parameters. The two of them are free parameters and the other one is determined by other two and observational data. If we take n and λ as free parameters, R_s is approximately proportional to λ^{-1} [23]. Note that n should be taken sufficiently large, $n \gtrsim 2$, if we want to obtain a noticeable effect for the density perturbation enhancement (see below). The model (3) can describe the accelerated expansion of the present Universe and it quickly approaches the ΛCDM model for redshift $z > 1$ if we take large n and λ. Strictly speaking, a term proportional to R^2 should be added to (3) to avoid the scalaron mass M_s exceeding the Planck mass for high, but not too high matter densities in the early Universe [21], as well as to exclude possible formation of an extra weak curvature singularity in the recent past which was found in [25, 26] (still there remains an open question what would occur instead of this singularity [27]). However, the coefficient of this term (usually written as $(6M^2)^{-1}$ where M coincides with the scalaron mass M_s in the regime when this term dominates other non-GR terms in (3)) should be very small in order not to destroy the standard evolution of the early Universe. Namely, either M should be larger than the Hubble parameter H at the end of inflation, or this term can drive inflation by itself if $M \approx 3 \times 10^{13}$ GeV [28, 29]. With the R^2 term, the functional form (3) which is responsible for the late time acceleration should also be reconsidered and changed for $R < R_s$ (including the region $R < 0$) that is needed for nonsingular evolution of the model just after the end of inflation. We study this point deeply in [30]. But if we are interested in the dynamics of the late time Universe only, we can neglect both the R^2 term and the accompanying modification for $R < R_s$. Hence, we use Eq. (3) as the low energy effective theory for the following.

The model (3) describes similar background expansion history with that of the ΛCDM model. Although the equation of state parameter for dark energy makes a phantom crossing at $z \sim 3$ [19, 23, 31], it does not change CMB spectrum significantly. On the other hand, fluctuations evolve differently. We define the metric perturbation by the following notation,

$$ds^2 = -(1 + 2\Phi)dt^2 + a^2(t)(1 - 2\Psi)\delta_{ij} dx^i dx^j. \tag{4}$$

We can derive the effective gravitational constant and the gravitational slip in $f(R)$ gravity by using subhorizon limit and quasi-static approximation:

$$\frac{k^2}{a^2}\Phi = -4\pi G_{\rm eff}(t, k)\rho\Delta, \tag{5}$$

$$\frac{\Psi}{\Phi} = \eta(t, k), \tag{6}$$

where

$$\frac{G_{\rm eff}(t, k)}{G} = \frac{1}{f'} \frac{1 + 4\frac{k^2}{a^2}\frac{f''}{f'}}{1 + 3\frac{k^2}{a^2}\frac{f''}{f'}}, \tag{7}$$

$$\eta(t, k) = \frac{1 + 2\frac{k^2}{a^2}\frac{f''}{f'}}{1 + 4\frac{k^2}{a^2}\frac{f''}{f'}}. \tag{8}$$

Here, Δ is the gauge-invariant comoving matter perturbation, and the prime denotes the derivative with respect to the Ricci curvature. Thus, in the quasi-GR regime when $f' \approx 1$, the effective gravitational constant can become up to 33% larger, independently of a detailed functional form of $f(R)$. This is the cause of the enhancement of perturbation growth involved.

As a result of the time and the scale dependences of these parameters, evolution of matter density fluctuation is different from that in ΛCDM model, namely, it is enhanced. It promotes formation of LSS [19, 21, 23, 32]. On the contrary, the light neutrinos suppress structure formation by free streaming. Therefore, $f(R)$ modification and neutrino masses play opposite roles on the growth of perturbation and thus the allowed range for the total neutrino mass is relaxed in $f(R)$ gravity, compared with the ΛCDM model [24]. We can apply this mechanism for the case of sterile neutrinos.

We have carried out Markov Chain Monte Carlo (MCMC) analysis for the ΛCDM model and $f(R)$ gravity with one or two sterile neutrinos. Practically, we neglect rest masses of three standard neutrino types (assuming that $\sum_{i=1}^{3} m_{\nu i} < 0.1$ eV) compared to those of one or two sterile neutrino types. We have modified the MGCAMB [33, 34], which provides the evolution of the modified growth of matter fluctuation by setting $G_{\rm eff}(t, k)$ and $\eta(t, k)$ as special parameterization, so that it allows to implement $f(R)$ gravity by adopting (7) and (8). We do not change the background evolution equations, *i.e.*, we keep that in the ΛCDM model, because the difference between the background evolution in the viable $f(R)$ model and the ΛCDM model is not significant (though it is not exactly zero). We used the above modified MGCAMB and CosmoMC [35, 36] to constrain the model parameters. The free parameters are the density parameter for the dark matter $\Omega_{\rm DM}h^2$, sound horizon angle $\theta_* \equiv 100 r_s(z_*)/D_A(z_*)$, massive neutrino ratio, f_ν,

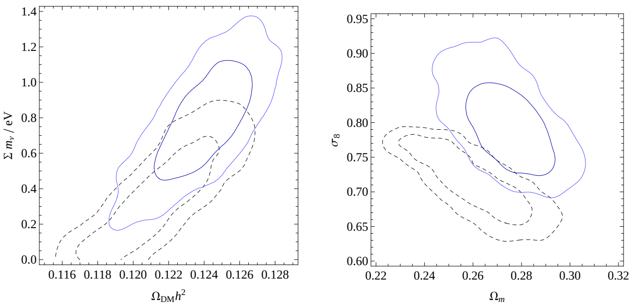

FIG. 1: 1 and 2σ contours of the sterile neutrino mass (left) and σ_8 (right) for the cases with three massless and one massive neutrinos in the ΛCDM model (dashed black) and $f(R)$ gravity (solid blue). $\chi^2_{\rm eff} = 3774.1$ and 3767.0, respectively.

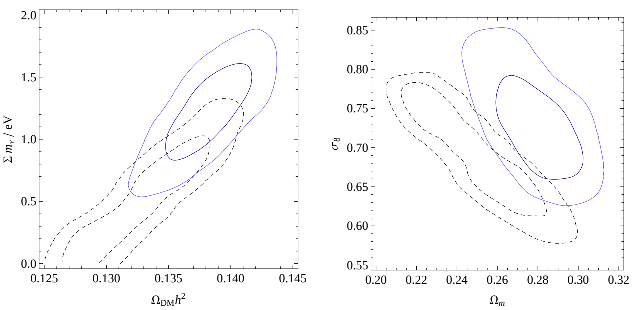

FIG. 2: 1 and 2σ contours for the sterile neutrino mass (left) and σ_8 (right) for the cases with three massless and two massive neutrinos in the ΛCDM model (dashed black) and $f(R)$ gravity (solid blue). $\chi^2_{\rm eff} = 3788.7$ and 3779.3, respectively.

which is the fraction of the energy density of dark matter in the form of massive neutrinos:

$$f_\nu \equiv \frac{\Omega_\nu h^2}{\Omega_{\rm DM} h^2} = \frac{1}{\Omega_{\rm DM} h^2}\frac{\sum m_\nu}{94.1{\rm eV}}, \tag{9}$$

and the amplitude of the $f(R)$ modification, λ. Here, $\Omega_{\rm DM}$ means the sum of the contribution from cold dark matter and massive neutrinos. The value $n = 2$ is chosen because it is the minimal integer value for which the scalaron mass M_s given by $M_s^2 = 1/3f''(R)$ in the quasi-GR regime is, on one hand, not much higher than the Hubble constant H_0 if estimated at the present cosmic background matter density $\rho_{m0} = 3\Omega_m H_0^2/(8\pi G)$ (see the value of the constant B_0 below which characterizes it quantitatively), and on the other hand, it is already sufficiently large for matter densities inside the Solar system (not speaking about those in laboratory experiments) to make scalaron heavy and unobservable even outside gravitating bodies similar to dilaton in the string theory. Indeed, for the functional form (3), $M_s \propto \rho_m^{n+1}$ in the quasi-GR regime for $\rho_m \gg \rho_{m0}$. Thus, here there is no necessity to consider the more subtle chameleon effect which can make scalaron unobservable even if it is light outside bodies (though heavy inside them).

Further, in order to have the future stable de Sitter stage, λ should be larger than 0.95. For $n = 2$ and $\lambda = 0.95$, the deviation index $B_0 = (f''/f')(dR/d\ln H)|_{t=t_0}$ is not too small nor too large, namely, $B_0 = 0.21$ [23]. This value is in agreement with the upper limit $B_0 < 0.4$ recently obtained in [37]. Contrary, the much more stringent upper limit obtained from cluster abundance in [38] is not applicable for our model because it was obtained while for a similar functional form of $f(R)$ introduced in [19], but for its parameter value characterizing the large-R behaviour which corresponds to $n = 0.5$ in Eq. (3). We fix the other cosmological parameters by the mean value constrained by WMAP7 data only. To constrain the free parameters, we used the observational data of CMB by WMAP7 [9] and matter power spectrum by SDSS DR7 [39].

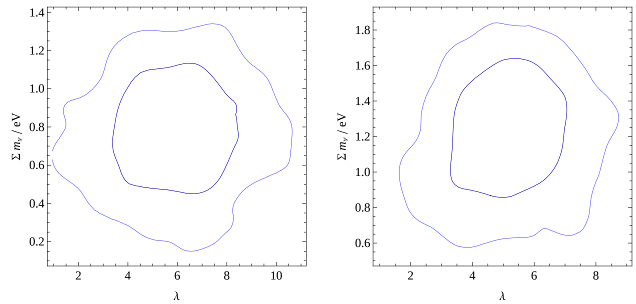

FIG. 3: 1 and 2σ contours for the sterile neutrino mass and $f(R)$ parameter λ for the case with three massless and one massive neutrinos (left) and three massless and two massive neutrinos (right). $\lambda > 0.95$ to guarantee the stability of the future de Sitter stage.

Figures 1 and 2 depict the contour plot for the case of one and two sterile neutrinos, respectively. In the ΛCDM model, the total neutrino mass is constrained in the sub-eV range at 2σ. On the other hand, it is allowed to take up to the order of 1 eV in $f(R)$ gravity. We also find a larger value of σ_8 with the mean value about 0.8 that expresses the enhancement of perturbations as we mentioned above. Comparing Figs. 1 and 2, it is seen that in the two sterile neutrino case the value of σ_8 is slightly smaller, about 5 %. This presents the possibility to distinguish these two scenarios when more exact data on σ_8 will be obtained, *e.g.*, from cluster abundance.

Figures 3 also suggest $f(R)$ gravity favors the total sterile neutrino mass around 1 eV with mild values of λ with which small-scale enhancement of fluctuations is appreciable.

Moreover, we find that in the presence of the massive sterile neutrino $f(R)$ gravity fits the cosmological data much better than the ΛCDM model. Indeed, for the case with a single sterile neutrino species the best-fit values are $m_\nu = 0.860$ eV, $\lambda = 5.72$, and $\chi^2_{\rm eff} = 3767.0$ for $f(R)$ gravity, and $m_\nu = 0.109$ eV and $\chi^2_{\rm eff} = 3774.1$ for the ΛCDM model. According to the Akaike Information Criteria (AIC) [40], if $\chi^2_{\rm eff}$ improves by 2 or more with a new additional fitting parameter, its incorporation is

justified. In this context, the performance of $f(R)$ gravity is very well improving $\chi^2_{\rm eff}$ by more than 7. In the case with two sterile neutrino species, the improvement of $\chi^2_{\rm eff}$ is even larger, 9.4.

Note that these are comparisons between the best-fit values for which the ΛCDM model gives a significantly smaller values of the sterile neutrino mass. Needless to say, the existence and the mass of the sterile neutrino is to be determined by ground-based experiments rather than cosmology which also depends on other factors that fix the evolution of the background and perturbed Universe in a complicated manner.

If, for example, future experiments fix the sterile neutrino mass at, say, $m_\nu = 1$ eV, then $f(R)$ gravity yields $\chi^2_{\rm eff} = 3767.3$ with the best-fit $\lambda = 5.97$, while the ΛCDM model yields $\chi^2_{\rm eff} = 3778.0$ significantly worse than the former. Moreover, ΛCDM model yields $\sigma_8 = 0.622 \pm 0.005$, which is in disagreement with observation [16, 39]. $f(R)$ gravity, on the contrary, also works well in this respect with $\sigma_8 = 0.76 \pm 0.02$ for this fixed mass of m_ν.

Thus we conclude that if a ~ 1 eV sterile neutrino is indeed established by ground-based experiments, that is in fact favored by a number of experiments now, then cosmological data strongly favor cosmology based on $f(R)$ gravity compared to the canonical ΛCDM model.

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